DETERMINATION THE EFFECT OF CIRCULAR HOLE ON THE STRESS CONCENTRATION FOR A PLATE SUBJECTED TO UNIAXIAL AND BIAXIAL STRESSES IN TENSION AND COMPRESSION STATUS

Najah Rustum Mohsin
Mechanical Techniques Department, Southern Technical University, Technical Institute - Nasiriya, Iraq

Younis Fakher Aoda
Mechanical Techniques Department, Southern Technical University, Technical Institute - Nasiriya, Iraq

Raheem Abd Sayel
Mechanical Techniques Department, Southern Technical University, Technical Institute - Nasiriya, Iraq
Abstract
This paper deals with investigation of applied stresses, plate length, plate width and the
diameter of the hole on the maximum stress for the plate with a one center hole and two holes
subjected to uniform uniaxial tension, uniaxial compression and biaxial tension stresses. The
maximum stresses are numerically calculated by finite element software ANSYS R.15 and
theoretically for different cases. The results show that the values of maximum stress in case
of uniaxial tension are greater than of biaxial tension and both of them greater than of
uniaxial compression. Furthermore, decreasing the applied stresses and hole diameter and
increasing the plate length and width lead to decreasing the value of maximum stresses.

KEY WORDS: Maximum stress, center hole, uniaxial tension, biaxial tension, uniaxial
compression, ANSYS R.15.

1. Introduction
Abrupt changes originated from irregularities in the distribution of stresses are known as
stress concentrators; these are presented for all types of stress, axial, bending or shear in the
presence of fillets, holes, grooves, keyways, splines, tool marks or accidental scrapes. The
inclusions or defects within the material over the surface also serve as "stress risers". The first
mathematical study on stress concentration was published shortly after 1900, with the aim of
working with other very simple different cases, experimental methods were developed to
measure local efforts. In recent years they have started using computer simulations based on
finite elements. Santos[1].

FEM is used by N. K. Jain [2] to study the distributions of stresses and deflection in
rectangular isotropic and orthotropic plates with central circular hole under transverse static
loading. S. P. Berlo [3] studied the effect of localized non-homogeneity in material property
on stress concentration. Two-dimensional infinite plane theory with both biaxial and uniaxial
far field loading was applied to problems with remote stress free holes, both circular and
elliptical. A. Santos [1] used ANSYS software to determine stress concentration factors in
flat plates with a central hole subjected to axial load. Effect of the geometry of hole on the
stress distribution around the hole was studied by D. Gunwant and J. P. Singh [4]. A
continuous elastic plate made up of steel with a central elliptical hole has been modeled and
descriptive with SOLID95 elements using ANSYS software. Zamanian et al. [5] were used
ABAQUS software to calculate the maximum stress for the SMC - R65 laminas with
circular/square hole under uniaxial loading. A comparison on stress concentration factor for randomly oriented discontinuous fiber laminas between the results obtained with photo elastic, Howland and Heywood formulations has been done for the plate with a circular hole. M. Mirzagoltabaroshan [6] used the ruling equation in mixed form by partial derivative in fourth order, numerical approach with finite difference model and numerical approach with ANSYS software to calculate the effect of hole and opening on stress concentration phenomenon and its coefficient changes in plates and the shells. G. C. Mekalke et al. [7] were used finite element method to study the effect of an initial stretching of a rectangular plate with a cylindrical hole on the stress and displacement distributions around it. They assumed that the initial stresses are caused by the uniformly stretching forces acting on the 2 opposite ends. A. Khechaiet at. [8] were calculated the stress concentration factors in cross-and-angle-ply laminated composite plates as well as in isotropic plates with single circular holes subjected to uniaxial loading. A quadrilateral finite element of four-node with 32 degrees of freedom at each node are used to evaluate the stress distribution in laminated composite plates with central circular holes. the transverse vibrations and the natural frequencies of rectangular plate with circular central hole are investigated by K. Torabi and A.R. Azadi [9] using Rayleigh-Ritz Method. The effect of the hole is taken into account by subtracting the energies of the hole domain from the total energies of the whole plate. A. A. Abd-Elhady [10] was studied the variations of stress and strain concentration factors for plate with small central notch, circular notch and double U-notch subjected to uniaxial and biaxial loading. The influence of the notch radius and plate thickness on the elastic stress and strain concentration factors has been studied. Stress concentration factors at the root of an elliptic hole in unidirectional functionally graded material plates under uniaxial and biaxial loads are studied by T. A. Enab [11] using ANSYS Parametric Design Language (APDL) to build the finite element models for the plates.

Consider the plate shown in Figure 1, loaded in tension by a force per unit area, consider that the outer dimensions of the plate are infinite compared with the diameter of the hole, 2a. It can be shown, from linear elasticity that the tangential stress throughout the plate is given by:

\[
\sigma_\theta = \sigma_0 \left[ 1 + \frac{a^2}{r^2} - \left( 1 + 3 \frac{a^4}{r^4} \right) \cos 2\theta \right] \quad \text{......................... (1)}
\]

The maximum stress is \( \sigma_\theta = 3\sigma \) at \( r = a \) and \( \theta = \pm 90^\circ \). Figure 2 shows how the tangential stress varies along the x and y axes of the plate. For the top (and bottom) of the hole, we see
the stress gradient is extremely large compared with the nominal stress, and hence the term stress concentration applies. Along the surface of the hole, the tangential stress is $-\sigma$ at $\theta = 0^\circ$ and $180^\circ$, and increases, as $\theta$ increases, to $3\sigma$ at $\theta = 90^\circ$ and $270^\circ$.

![Circular Hole in a Plate](image1)

**Figure 1: Circular Hole in a Plate**

**Figure 2: Tangential Stress Distribution**

Loaded in Tension for $\theta = 0^\circ$ and $90^\circ$

The static stress concentration factor in the elastic range, $K_t$, is defined as the ratio of the maximum stress, $\sigma_{\text{max}}$, to the nominal stress, $\sigma_{\text{nom}}$. That is,

$$K_t = \frac{\sigma_{\text{max}}}{\sigma_{\text{nom}}} \quad \text{.......................... (2)}$$

For the infinite plate containing a hole and loaded in tension, $\sigma_{\text{nom}} = \sigma$, $\sigma_{\text{max}} = 3\sigma$, and thus $K_t = 3$.

The analysis of the plate in tension with a hole just given is for a very wide plate (infinite in the limit). As the width of the plate decreases, the maximum stress becomes less than three times the nominal stress at the zone containing the hole. Young and Budynas[12]. Figure 3 shows the stress distribution for a plate with a central hole under uniaxial tension stresses.

![Stress Distribution](image3)

**Figure 3: Stress Distribution for a Plate in Tension Containing a Centrally Located Hole**
2. Materials and Methods

Circular hole in finite-width plate specimens subjected to uniform uniaxial and biaxial stresses in tension and compression status as shown in Figures 4 a, b and c are studied to determine the stress concentration using theoretical and numerical solutions.

![Figure 4: Circular Hole in Finite Width Specimens Subjected to](image)

- **a)** Uniform Uniaxial Tension
- **b)** Uniform Uniaxial Compression
- **c)** Uniform Biaxial Tension

2.1 Specimens Material

The material of plate specimens is a Carbon steel with modulus of elasticity =202E-3 MN/m², poison’s ratio = 0.292 and density = 7820 Kg/m³[13]. The models of ANSYS plate specimens with elements, nodes and boundary conditions as shown in Figure 5.

![Figure 5: ANSYS Models are used in the Solutions.](image)
2.2 Theoretical Solution

Maximum stresses are calculated theoretically using the following formulas [12]

\[
\sigma_{\text{nom}} = \frac{b}{b-2a} \times \sigma \\
K_t = 3.00 - 3.13 \left( \frac{2a}{b} \right) + 3.66 \left( \frac{2a}{b} \right)^2 - 1.53 \left( \frac{2a}{b} \right)^3 \\
\sigma_{\text{max}} = \sigma_{\text{nom}} \times K_t
\]  

(3) \hspace{1cm} (4) \hspace{1cm} (5)

2.3 Numerical Solution

Since the evolution of the term finite element by Clough in 1951, there have been significant developments in finite element method. In this paper, Maximum stresses numerically calculated using finite element software ANSYS R15 with Plane183 element as a discretization element. Due to symmetry, only a quarter and half sectors of the plate is modeled for case 1 and 2, respectively as shown in Table 1. Mesh generation is used for node and element creation and the area around the hole is remodeled with a fine mesh.

2.4 Plane183 Element Description

PLANE183 is used in this paper as a discretization element. It is defined by 8 nodes (I, J, K, L, M, N, O, P when quadrilateral element) or 6 nodes (I, J, K, L, M, N when triangle element) having two degrees of freedom (UX, UY) at each node (translations in the nodal X and Y directions). ANSYS help [14]. The geometry, node locations, and the coordinate system for this element are shown in Figure6.
2.5 The Studied Cases

Many cases (Table 1) are studied theoretically and numerically to explain the effect of circular hole in a finite plate on the maximum stresses.

Table 1: The cases studied with Solutions, Parameters, ANSYS Models and Figures

<table>
<thead>
<tr>
<th>Number</th>
<th>Holes</th>
<th>Solution Type</th>
<th>Stresses Type</th>
<th>Unchanged parameters (cm and N/cm²)</th>
<th>Changed Parameters (cm and N/cm²)</th>
<th>ANSYS Model</th>
<th>Figure No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Theoretical &amp; Numerical</td>
<td>Uniaxial Ten.</td>
<td>l=30, b=30, 2a=5</td>
<td>σ = 1000, 1200, 1400, 1600, 1800, 2000</td>
<td>Figure(5 b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Theoretical &amp; Numerical</td>
<td>Uniaxial Ten.</td>
<td>σ =1000, b=30, 2a=5</td>
<td>1 = 15, 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Theoretical &amp; Numerical</td>
<td>Uniaxial Ten.</td>
<td>σ =1000, l=30, 2a=5</td>
<td>b = 15, 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Theoretical &amp; Numerical</td>
<td>Uniaxial Ten.</td>
<td>σ =1000, b=30, l=30</td>
<td>a=1, 2, 3, 4, 5, 6, 7</td>
<td>Figure(5 b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>l=30, b=30, 2a=5</td>
<td>σ = 1000, 1200, 1400, 1600, 1800, 2000</td>
<td>Figure(5 b, d, f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, b=30, 2a=5</td>
<td>1 = 15, 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 b, d, f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, l=30, 2a=5</td>
<td>b = 15, 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 b, d, f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, b=30, l=30</td>
<td>2a=2, 4, 6, 8, 10, 12, 14</td>
<td>Figure(5 b, d, f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>l=30, b=30, 2a=5</td>
<td>σ = 1000, 1200, 1400, 1600, 1800, 2000</td>
<td>Figure(5 h, j, l)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, b=30, 2a=5</td>
<td>1 = 15, 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 h, j, l)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, l=30, 2a=5</td>
<td>b = 20, 25, 30, 35, 40, 45</td>
<td>Figure(5 h, j, l)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Numerical</td>
<td>Uniaxial Ten., Uniaxial Comp. &amp; Biaxial Ten.</td>
<td>σ =1000, b=30, l=30</td>
<td>2a=1, 2, 3, 4, 5, 6, 7, 8</td>
<td>Figure(5 h, j, l)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3. Results and Discussions

3.1 Theoretical and Numerical Compression

Figures 7-10 illustrate the numerical and theoretical variations of $\sigma_{\text{max}}$ with different values of $\sigma$, $l$, $b$ and $2a$, respectively for the plate subjected to uniaxial tension. From these figures, it can be seen that there are a small difference occurs between the two solutions with variation of $\sigma$ and $b$ while increasing $2a$ and decreasing $l$ lead to increase the difference between the two solutions.

Figure 7: Variation of maximum stress with applied uniaxial tension stress theoretically and numerically

Figure 8: Variation of maximum stress with the length of plate theoretically and numerically

Figure 9: Variation of maximum stress with the width of plate theoretically and numerically

Figure 10: Variation of maximum stress with the hole diameter theoretically and numerically
4. Numerical Solution

4.1 One Hole

The variation of $\sigma_{\text{max}}$ for the plate with center hole subjected to uniaxial tension, uniaxial compression and biaxial tension stresses with different values of $\sigma$, $l$, $b$ and $2a$ are shown in Figures 11-14, respectively. In all mentioned figures, it is found that the values of $\sigma_{\text{max}}$ in case of uniaxial tension are greater than of biaxial tension and both of them greater than of uniaxial compression except in case when increase the diameter of hole, at this point we show that the $\sigma_{\text{max}}$ in uniaxial compression exceed the biaxial tension. Furthermore, it is clear that $\sigma_{\text{max}}$ directly proportional with the applied stress and the hole diameter but reversal proportional with the length and width of plate.

Figure 11: Variation of maximum stress with the applied stress at uniaxial tension, uniaxial compression and biaxial tension for one hole

Figure 12: Variation of maximum stress with the plate length at uniaxial tension, uniaxial compression and biaxial tension for one hole

Figure 13: Variation of maximum stress with the plate width at uniaxial tension, uniaxial compression and biaxial tension for one hole

Figure 14: Variation of maximum stress with the hole diameter at uniaxial tension, uniaxial compression and biaxial tension for one hole
4.2 Two Holes

Figures 15-18 explain the variation of maximum stress with different values of $\sigma$, $l$, $b$ and $2a$ for the plate with two holes subjected to uniaxial tension, uniaxial compression and biaxial tension stresses. From these figures, it is found that the behavior of the maximum stress in this case is the same with the previous case but small difference in the values between them.

Over and above, Figures 19-21 graphically illustrated $\sigma_{\text{Von-Mises}}$ counter plots for different cases reported in Table 2.
Table 2: Figures number for Von-Misses Stresses Counter Plots with ANSYS Models and Parameters

<table>
<thead>
<tr>
<th>Figure No.</th>
<th>Holes</th>
<th>ANSYS Model Figure</th>
<th>Stresses Type</th>
<th>Unchanged parameters (cm and N/cm²)</th>
<th>Changed Parameters (cm and N/cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>19a, c, e</td>
<td>1</td>
<td>5a, c, e</td>
<td>Uniaxial Ten.,</td>
<td>σ = 1800, b=30, l=30, 2a=5</td>
<td>applied stresses</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Uniaxial Comp., Biaxial Ten.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19b, d, f</td>
<td>2</td>
<td>5g, I, k</td>
<td>Uniaxial Ten.,</td>
<td>σ = 1800, b=30, l=30, 2a=5</td>
<td>applied stresses</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Uniaxial Comp., Biaxial Ten.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20a, c, e, g</td>
<td>1</td>
<td>5b</td>
<td>Uniaxial Ten.</td>
<td>σ = 1000, b=30, l=30, 2a=2, 6, 10, 14</td>
<td></td>
</tr>
<tr>
<td>20b, d, f, h</td>
<td>2</td>
<td>5h</td>
<td>Uniaxial Ten.</td>
<td>σ = 1000, b=30, l=30, 2a=2, 6, 10, 14</td>
<td></td>
</tr>
<tr>
<td>21a, c, e, g</td>
<td>1</td>
<td>5b</td>
<td>Uniaxial Ten.</td>
<td>σ = 1000, b=30, l=30, 2a=2, 6, 10, 14</td>
<td></td>
</tr>
<tr>
<td>21b, d, f, h</td>
<td>1</td>
<td>5f</td>
<td>Biaxial Ten.</td>
<td>σ = 1000, b=30, l=30, 2a=2, 6, 10, 14</td>
<td></td>
</tr>
</tbody>
</table>
Figure 19: Von-Misses CounterPlots for Full ANSYS Models to Plates with 1 and 2 Holes
Figure 20: Von-Misses CounterPlots for Quarter and Half ANSYS Models to Plates with 1 and 2 Holes
Figure 21: Von-Misses CounterPlots for Quarter ANSYS Models to Plates with 1 Hole under Uniaxial and Biaxial Tension
5. Conclusion

The main conclusions of this work are reported below

1. The values of maximum stress in case of uniaxial tension are greater than of biaxial tension and both of them greater than of uniaxial compression.

2. Increasing the applied stresses and hole diameter and decreasing the plate length and width lead to increasing the value of maximum stresses.

3. The behaviour of the maximum stress for plates with one center hole is the same with the plates with two holes but there is a sensitive difference in the values between them.
References


ANSYS help.