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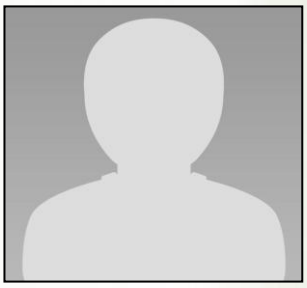
"A case study on Performance Analysis of Government, Aided and Unaided Schools in Kerala Using DEA"



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Introduction

Data Envelopment Analysis (DEA) is a methodology based upon linear programming employed for assessing the relative performance of a set of firms that use a variety of identical inputs to produce a variety of identical outputs. The principles of DEA date back to Farrel(1957), and the recent works by Charnes et al.(1978),Norman and Stoker(1991), Cooper et al.(2000, M. Khodabakhshi (2010). DEA is a linear program technique for measuring the performance efficiency of organizational units like banks, mutual fund, police stations, hospitals, tax offices, defence bases, insurance companies, schools, libraries and university departments which are termed Decision Making Units (DMUs). DEA can handle multiple inputs and multiple outputs as opposed to other techniques such as ratio analysis or regression. By identifying the "peers" for organizations that are not observed to be efficient, it provides a set of potential role models that an organization can look into, in the first instance, for ways of improving its operations. It is a useful tool for examining the efficiency of government service providers. DEA (in conjunction with other measures) be explored to increase our understanding of public sector performance and potential ways of improving it. The performance of a unit is evaluated by comparing its performance with the best performing units as an efficiency frontier and thus DEA is called frontier analysis.

Data preparation

The performance analysis on three categories is considered –Government School, Private Aided School, and Private Unaided School. The schools were finalized considering Judgment sampling depending on the moderate performance of schools, history of schools, and opinion of locals about teachers and students, parent's status. Three Govt Schools,(G1,G2,G3), 3 private Aided Schools (A1,A2,A3) and 5 Unaided Schools (U1,U2,U3,U4U5) were selected to form sample Frame. Proportional to the strength of students, using stratified random sampling, allowing 8% of variation on opinion, the sample size is determined for 7^{th}, 9^{th}, 11^{th} standard classes of each school. A questionnaire of 20 questions is prepared separately for students and their parents subjective to the inputs and outputs predesigned for the study. Questions for students had 5 point scale responses on time of teaching and reading, conveniences, infrastructure, library, laboratory, travel facility, examination, coeducation, syllabus, teachers quality , awareness, drug and drink habits likes and dislikes of students etc. Teacher assessment is done on the regularity, teaching aids, intimacy, counselling, extension activities …. Parents data includes financial aids, teacher –parent relation, homework, ability building, economic and social condition etc. Average responses and standard deviation of 5 point scale data is used for the preparation of input and outputs for each School called DMUs

Using the prepared data on average and SD, appropriate factors were detected for the responses of Students and Parents using factor analysis. Factor Reduction is applied by fixing the Chronbach's Alpha greater than 0.4 for each revised factors and thus 4 inputs and 3 outputs were determined. The following are the designed inputs and outputs for each DMU – School – for applying DEA

1) Inputs:
1) Percentage of available Physical Facilities (PPF)
2) Percentage of available Ancillary Facilities (PAF)
3) Teacher's Qualification Index (TQI)
4) Education-Occupation Index of parents (EOI)

2) Outputs:
1) Students' Satisfaction (SS)
2) Students' participation in extra curricular Activities (SEA)

3) Social Awareness in Students (SAS)

1. Elementary Average Analysis

The following are some of the primary results derived from the input and output of the study
Table: 1 p values between various Factors of significant differences

	(I)PAF	(I)EQI	(I)TQI
(I)PPF	0.0163<0.05	9.69847E-08<0.05	0.0552>0.05
(I)PAF		3.15801E-07<0.05	0.3243>0.05
(I)EQI			2E-07<0.05

1. There is significant difference between Means of Factors –PPF Vs PAF and PPF Vs EQI

2. There is significant difference between Means of Factors –PAF Vs EQI and PAF Vs PPF

3. There is significant difference between Means of Factors –EQI Vs PPF and EQI Vs PAF & EQI Vs TQI

4. The Mean EQI is significantly different between Government school and Aided School. (pvalue=0.0042<0.05) and Government school and UnAided School. (pvalue=0.0022<0.05)

5. There is significant difference between Means of the Factor- PAF between Male and Female of Government School and Total School. (pvalue=0.0199<0.05) and (pvalue=0.0477<0.05)

6. There is significant difference between Means of the Factor- EQI among Males and Females of Government Vs Unaided School. (pvalue=0.0111<0.05) and (pvalue=0.0049<0.05)

7. There is significant difference between Means of the Factor- PAF between Students and Parents of Aided School (pvalue=0.0353<0.05)

8. There is significant difference between Means of the Factor- EQI between Students and Parents of Aided School, Unaided School and Total (pvalue=0.0147<0.05, pvalue=0.0013<0.05, pvalue=0.0003<0.05)

9. There is significant difference between Means of the Factor- SS between Students and Parents of Aided School, Unaided School and Total (pvalue=0.0136<0.05, pvalue=0.0012<0.05, pvalue=0.0004<0.05)

10. There is significant difference between Means of the Factor- PAF of Parents of Govt and Aided School, Govt and Unaided School. (pvalue=0.0224<0.05, pvalue=0.0267<0.05)
Thus PAF and EQI are the major incoming Factors influencing the difference in the performance of Govt and Aided School or Govt and Unaided School.

2. Factor efficiency comparison using Frontier Analysis

The performance of DMUs is assessed using the concept of efficiency or productivity, and finding the optimum of the ratio of total outputs to total inputs applied in DEA.

Efficiency = Output / Input

Efficiencies estimated using DEA are relative, i.e., relative to the best performing DMU (or DMUs if there is more than one best performing DMUs).The best performing DMU is assigned an efficiency score of unity or 100%, and the performance of other DMUs vary, 0 and 100 per cent relative to this best performance.

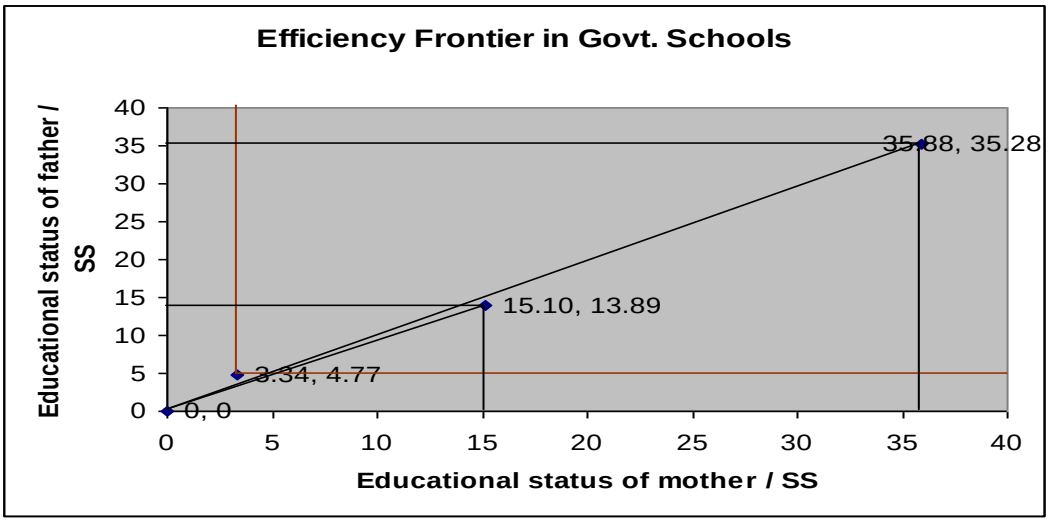
For single input and single output, finding ratio for efficiency is simple and the most efficient DMU is one which uses minimum input to get maximum output. If there is more than one input, one way to tackle the problem of interpreting different ratios, is graphical analysis. The efficiency analysis of educational status- EOI among the parents (inputs) with the students' satisfaction (SS) output- is derived by Frontier Analysis.

Table 2: Frequency and efficiency ratio of Education Qualifications

Govt School	**Input**		**Output**	**Ratio 1**	**Ratio 2**
Educational Qualification	Mother	Father	SS	Mother	Father
High school (1)	60	59	1.67	35.88	35.28
Graduates (2)	25	23	1.66	15.10	13.89
Post graduates (3)	7	10	2.10	3.34	4.77

The three efficiency ratios are plotted in a XY plane as in Figure 1. The minimum input is more efficient so that it will fall nearer to the origin. Hence Post Graduates seems to be more efficient. Efficiency Frontier is the line joining the more efficient point to the vertical and horizontal axis (red lines). It represents the optimum performance of all possibilities and all that is not on the frontier should try to achieve. Post graduate is close to the frontier and considered to be 100% efficient. High School and Graduates are inefficient as it doesn't lie on the frontier.

Figure 1 : Graphical Method to detect efficiency Frontier



Relative efficiency = Best possible performance / Actual performance

The relative efficiency of High School educated parents compared Postgraduate parents on Students Satisfaction is 14% (from the frontier figure) and 35% respectively. Similar analysis is done for Aided and Unaided schools. It is found that in both cases PG parents is more efficient than others. For Aided schools the relative efficiencies of graduate and high school parents educated parents is 49% and 23% respectively. For unaided school respective relative efficiencies are 32% and 10% respectively.

3. School efficiency comparison using DEA

When there is more than two inputs or more than one output frontier analysis leads to DEA and so the Efficiency analysis of schools was performed using DEA. The model used is input oriented BCC (Banker, Charnes and Cooper) model, which satisfy assumptions and performing good results regarding school data .The inputs are PPF, PAF and EOI for 11 schools and output is the lone SS finalized after confirmatory factor analysis . With the help of DEA Solver, analysis is done and the corresponding tables and graphs were obtained.

Table:3 Average input and output factors in the BCC I DEA Model .

	DMU	G1	G2	G3	A1	A2	A3	U1	U2	U3	U4	U5
Input	PPF	1.83	1.90	1.44	1.70	1.80	1.57	1.51	1.54	1.86	1.60	1.55
	PAF	1.99	1.81	1.79	1.77	1.88	1.80	1.96	1.86	1.81	1.62	1.69
	EOI	2.26	2.20	2.82	3.92	3.54	3.82	3.32	3.97	3.45	4.19	3.97
Output	SS	1.84	1.83	1.51	1.79	1.91	1.82	1.83	1.95	1.81	1.77	1.74

By the BCC model LPP for G1 is: <G1> Mini θ_B

Sub to

$1.83\ \theta_B - 1.83\lambda_{G1} - 1.90\lambda_{G2} - 1.44\lambda_{G3} - 1.70\lambda_{A1} - 1.80\lambda_{A2} - 1.57\lambda_{A3} - 1.51\lambda_{U1} - 1.54\lambda_{U2} - 1.86\lambda_{U3} - 1.60\lambda_{U4} - 1.55\lambda_{U5} \geq 0$

$1.99\ \theta_B - 1.99\lambda_{G1} - 1.81\lambda_{G2} - 1.79\lambda_{G3} - 1.77\lambda_{A1} - 1.88\lambda_{A2} - 1.80\lambda_{A3} - 1.96\lambda_{U1} - 1.86\lambda_{U2} - 1.81\lambda_{U3} - 1.62\lambda_{U4} - 1.69\lambda_{U5} \geq 0$

$2.26\ \theta_B - 2.26\lambda_{G1} - 2.20\lambda_{G2} - 2.82\lambda_{G3} - 3.92\lambda_{A1} - 3.54\lambda_{A2} - 3.82\lambda_{A3} - 3.32\lambda_{U1} - 3.97\lambda_{U2} - 3.45\lambda_{U3} - 4.19\lambda_{U4} - 3.97\lambda_{U5} \geq 0$

$1.84\lambda_{G1} + 1.83\lambda_{G2} + 1.51\lambda_{G3} + 1.79\lambda_{A1} 1.91\lambda_{A2} + 1.82\lambda_{A3} + 1.83\lambda_{U1} + 1.95\lambda_{U2} + 1.81\lambda_{U3} + 1.77\lambda_{U4} + 1.74\ \lambda_{U5} \geq 1.84$

$e\ (\lambda_{G1} + \lambda_{G2} + \lambda_{G3} + \lambda_{A1} + \lambda_{A2} + \lambda_{A3} + \lambda_{U1} + \lambda_{U2} + \lambda_{U3} + \lambda_{U4} + \lambda_{U5}) = 1, \quad \lambda \geq 0$, where θ_B is a scalar.

Similar LPPs are formulated for rest of the DMUs. Solving this problem gives the optimum solution for θ_B and also the corresponding values of the parameters (λ^*, s^-, s^+), where s^+ and s^- are corresponding slack and surplus variables. For an inefficient DMU there exists a reference set, E_0 based on λ^* such that $E_0 = [\ j / \lambda_j^* > 0\ ]$; $j \in 1,2,\ldots n$.

Table: 4 Score, Rank and Reference sets

No.	DMU	Score	Rank	Reference Set E_0 (λ^*>0)							
4	A1	0.96	10	G2	0.1672	G3	0.0468	U5	0.1646	U3	0.62
5	A2	0.97	9	G2	0.2983	U5	0.6616	U3	0.0401		
6	A3	0.99	8	G4	0.0387	G3	0.1647	U5	0.5106	U3	0.29
9	U4	0.95	11	G5	0.4446	U5	0.0939	U3	0.4614		

From the table, DEA score, Rank and Reference set, λ^* for each DMU is shown and it can be used to realize the efficiency for inefficient DMUs. Example: A2-the Aided school 2- is found to be inefficient. The reference sets for A2 are G2, U5, U3 with $\lambda_{G2}^* = 0.298$, $\lambda_{U5}^* = 0.662$, $\lambda_{U3}^* = 0.04$ shows the proportions contributed by them to evaluate A2 to reach efficiency. Hence A2 is technically inefficient and can be achieved the efficiency by practicing the above proportion factors of defined efficient reference schools. Similarly other aided and unaided schools can be interpreted to reach optimum efficiency.

Table: 5 Slack and Surplus of DMUs wrt to Score

No.	DMU	Score	Excess PPF	Excess PAF	Excess EQI	Shortage SS
1	G1	1	0	0	0	0
2	G2	1	0	0	0	0
3	G3	1	0	0	0	0
4	A1	0.96	0	0	0	0
5	A2	0.97	0.1042	0	0	0
6	A3	0.99	0	0	0	0
7	U1	1	0	0	0	0
8	U2	1	0	0	0	0
9	U3	0.95	0.0414	0	0	0
10	U4	1	0	0	0	0
11	U5	1	0	0	0	0

'Slack' shows the input excess s^- and output shortfall s^+ for each DMU. For example DMU A2 and U3 shows an excess input in PPF (0.1) and (0.04) respectively. The DMU without any excess of inputs and shortfalls of outputs is considered as efficient so that 7 DMUs are efficient –All Govt and all but one unaided schools are efficient. But all aided schools are inefficient. The removal of inefficiency is achieved by reducing the inputs PPF, PAF, EOI by 8.37%, 2.59% and 2.59% respectively. In fact, based on the reference sets and λ^*, the input and output values can be modified to bring A2 in to efficient status

0.97 × (input PPF of A2) = [0.29 × (input PPF of G2) + 0.66 × (input PPF of U5) + 0.04 × (input PPF of U3)]+0.01 reaching to 8.37% reduction in PPF .

Similarly for the remaining inputs and output, the same equation can be used and deficiencies can be rectified

Table: 6 DMU according to Rank and Score

Rank	DMU	Score	Rank	DMU	Score
1	U1	1	1	U4	1
1	G1	1	8	A1	0.99
1	G2	1	9	A2	0.97
1	G3	1	10	A3	0.96
1	U2	1	11	U5	0.95
1	U3	1			

The above table indicates the order of efficient schools wrt the input and output using BCC - Input Model. One Unaided (specifiable) is the optimum efficient model with respect to minimum input and the next three are Govt schools followed by three Unaided Schools. Three Aided Schools and one Unaided school lacking efficiency from 1% to 5%. It is observed that out of 11 DMUs 7 are efficient and 4 are inefficient. i.e., Aided schools are inefficient compared to Government and Unaided.

Table: 7 Projection of in efficient DMU

No	DMU	Score	Projec Tion	Difference	% diff	No	DMU	Score	Projection	Difference	% diff
4	PPF	1.702	1.632	-0.0693	-4.07%	6	PPF	1.571	1.555	-0.0161	-1.02%
	PAF	1.774	1.701	-0.0722	-4.07%		PAF	1.797	1.779	-0.0184	-1.02%
	EQI	3.917	3.757	-0.1594	-4.07%		EQI	3.815	3.776	-0.0391	-1.02%
	SS	1.794	1.794	0	0.00%		SS	1.82	1.82	0	0.00%
5	A2	0.974				9	U5	0.952			
	PPF	1.804	1.653	-0.1509	-8.37%		PPF	1.859	1.729	-0.1306	-7.03%
	PAF	1.882	1.834	-0.0488	-2.59%		PAF	1.815	1.728	-0.0870	-4.80%
	EOI	3.544	3.452	-0.0918	-2.59%		EQI	3.45	3.285	-0.1654	-4.80%
	SS	1.906	1.906	0	0.00%		SS	1.81	1.81	0	0.00%

The 'Projection Table provide the deviations of each DMU Factors from the efficient frontier factors for the model BCC -(BCC-Input)

The BCC-I projection achieved by the input factors corresponding to PPF, PAF, EOI of DMU A2 are

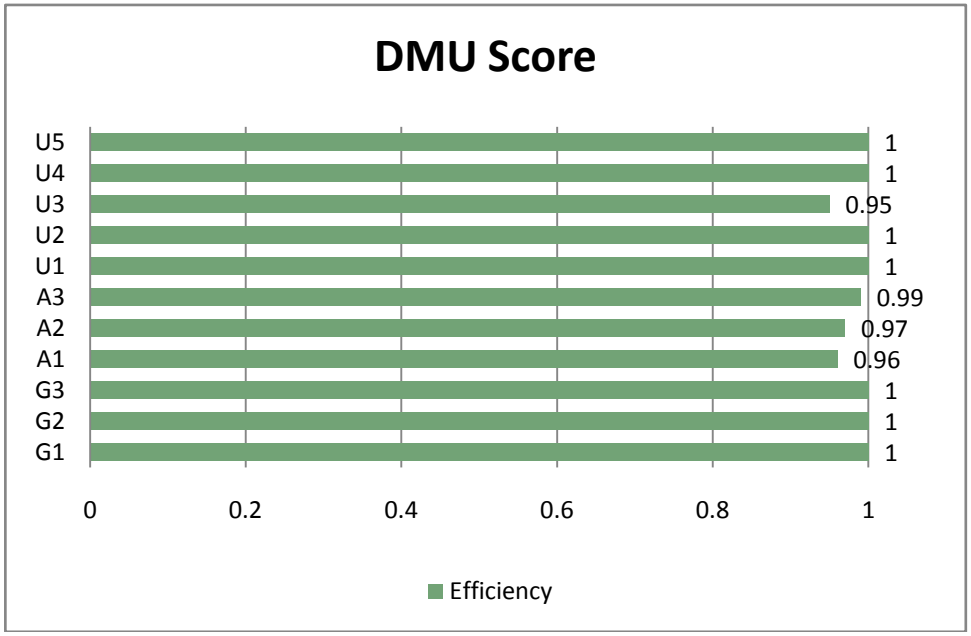
$x^{\wedge}_{PPF} = \theta^{*} \times x_{PPF} = 0.974 \times 1.804 = 1.65$ (8.37% reduction)

$x^{\wedge}_{PAF} = \theta^{*} \times x_{PAF} = 0.974 \times 1.882 = 1.83$ (2.59% reduction)

$x^{\wedge}_{EOI} = \theta^{*} \times x_{EOI} = 0.974 \times 3.544 = 3.45$ (2.59% reduction)

No projection is found for the output SS.

Figure 2: The efficient and inefficient Schools



4. Summary Outcomes of DEA

DEA model = BCC-I

No. of DMUs = 11
No. of Input items = 3 Input(1) = PPF Input(2) = PAF Input(3) = EQI
No. of Output items = 1 Output(1) = SS

No. of DMUs with inappropriate Data	**0**
No. of evaluated DMUs	**11**
Average of scores	**0.989**
No. of efficient DMUs	**7**
No. of inefficient DMUs	**4**

Correlation	PPF	PAF	EQI	SS
PPF	1	0.2293	-0.4533	0.4399
PAF	0.2293	1	-0.5488	0.3803
EQI	-0.4533	-0.5488	1	0.1672
SS	0.4399	0.3803	0.1672	1

The correlation table shows the correlation shows that there is significant correlation for PPF with PAF and SS and negative correlation with EQI .Similarly PAF is correlated with PPF and SS and negatively with EQI. EQI is slightly correlated with SS

Many other combinations of inputs and outputs are also tried to assess the efficiency of Govt, Aided and Unaided schools by considering all inputs along with one or two outputs in terms of sex of students or parent –student combination. The following are some important observations found and in some cases no relevant results of efficient schools are attained.

Results

Case1 Four inputs and Three outputs –BCC Model

1. Based on the output SEA there are 6 efficient and 5 inefficient DMUs. 4 Aided and one Unaided schools are seen inefficient.
2. Based on the output SS, 3 Aided and one Unaided schools are inefficient with respect to Govt. schools
3. Compared with outputs SS and SAS, DMUs A1 and U4 are seen inefficient.
4. Compared with outputs SEA and SAS, 4 DMUs namely A1, A3, U2 and U5 are inefficient.

Case 2: **8 inputs and 6 outputs of male and female students-BCC Model on Govt and Unaided schools**

1. When inputs are compared with single output SAS, 2 DMUs namely U4 and U5 are seen to be inefficient.
2. Compared with single output SS, DMU U4 is seen to be inefficient.
3. Compared with single output SEA, 2 DMUs namely U2 and U5 are seen to be inefficient.
4. Compared with the outputs SS and SEA, DMU U4 is seen to be inefficient.
5. Compared with outputs SEA and SAS, U5 is found to be inefficient.
6. Compared with output SS and SAS, U4 is seen to be inefficient.

Case 3: **8 inputs and 6 outputs of Students and Parents -BCC Model on Govt and Aided schools**

1. When inputs are compared with single output SS, G1 is seen to be inefficient.
2. Compared with single output SEA of students, 3 DMUs namely A2, A3 and G2 are seen to be inefficient.

3. Compared with single output SAS of students, 4 DMUs are seen to be inefficient including all Aided schools and one G1.
4. Compared with single output SS of students, 5 DMUs including 2 Govt. and 3 Aided are seen to be inefficient.
5. Compared with single output SS from parents, 1 DMU namely G1 is seen to be inefficient.
6. Compared with the Outputs SEA and SAS of students, 2 DMUs from the aided schools are seen to be inefficient.

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